

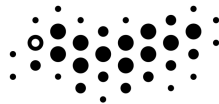
**Saint Petersburg National Research University of Informational
Technologies, Mechanics, and Optics**

**MATHEMATICAL CHALLENGE
OF QUANTUM TRANSPORT
IN NANOSYSTEMS -
PIERRE DUCLOS WORKSHOP**

International Conference

Saint Petersburg, September 9 – 11, 2015

Book of Abstracts



ITMO UNIVERSITY

**Saint Petersburg
2015**

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- Andrea Posilicano (Como, Italy)
- Silvestro Fassari (Locarno, Switzerland)
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- Andrea Mantile (Reims, France)
- Jaroslav Dittrich (Rez, Czech Republic)

THE MAIN TOPICS OF THE CONFERENCE:

Spectral theory
Scattering
Quantum transport
Quantum communications and computations

Conference Program

Pierre Duclos Workshop

September 09, 2015

9³⁰ – 10³⁰ *Registration (Pierre Duclos Workshop and Young Researches Symposium)*

10³⁰ – 10⁴⁰ *Opening*

Chairman: I.Popov

10⁴⁰ – 11⁴⁰ **Jussi Behrndt** (*Graz, Austria*)

Self-adjoint realizations of the Laplacian on bounded Lipschitz domains

11⁴⁰ – 12¹⁰ *Coffee*

Chairman: J.Behrndt

12¹⁰ – 13¹⁰ **Jaroslav Dittrich** (*Rez near Prague, Czech Republic*)

Strong delta-interaction supported by a surface

13¹⁰ – 14³⁰ *Lunch*

Chairman: J.Dittrich

14³⁰ – 15³⁰ **Andrea Mantile** (*Reims, France*)

Selfadjoint elliptic operators with boundary conditions on not closed hypersurfaces

15³⁰ – 16³⁰ **Andrea Posilicano** (*Como, Italy*)

Limiting absorption principle and factorized formulae for self-adjoint Laplace operators with boundary conditions on hypersurfaces

16³⁰ – 17⁰⁰ *Coffee*

Young Researchers Symposium-1 (YRS-1)

Chairman: I.Popov

17⁰⁰ – 17¹⁰ **Sergey Krasavin** (*Dubna, Russia*)

Effect of Stone-Wales defects on the thermal conductivity of graphene

17¹⁰ – 17²⁰ **Evgeniy Alexandrov** (*St. Petersburg, Russia*)

Nudged Elastic Band method for systems with constraints

17²⁰ – 17³⁰ **Pavel Smirnov** (*St. Petersburg, Russia*)

On the discrete spectrum of the Dirac operator on the branching chain quantum graph

17³⁰ – 17⁴⁰ **Ilya Makeev** (*St. Petersburg, Russia*)

Benchmark solutions for Stokes flow through nanostructures.

17⁴⁰ – 17⁵⁰ **Anton Boitsev** (*St. Petersburg, Russia*)

Boundary triples approach to extensions of operator tensor products

September 10, 2015

Chairman: A. Posilicano

10⁰⁰ – 11⁰⁰ **Claudio Cacciapuoti** (*Como, Italy*)

Singular limit of a Schrödinger equation with nonlocal scaled short-range nonlinearity

11⁰⁰ – 11³⁰ *Coffee*

Chairman: C. Cacciapuoti

11³⁰ – 12³⁰ **Adil Yafyasov** (*St. Petersburg, Russia*), Boris Pavlov (*Auckland, New Zealand*)

Size quantization and the resonance concept of the low-threshold emission

12³⁰ – 13³⁰ **Boris Pavlov** (*Auckland, New Zealand*), V. Flambaum (*Sydney, Australia*), G. Martin (*Auckland, New Zealand*)

Resonance mechanism of Earthquakes

13³⁰ – 14³⁰ *Lunch*

Chairman: B. Pavlov

14³⁰ – 15³⁰ **Pavel Stovicek** (*Prague, Czech Republic*)

A construction of the propagator for two Aharonov-Bohm solenoids in a uniform magnetic field

15³⁰ – 16⁰⁰ **Alexander Alodjants** (*Vladimir and St. Petersburg, Russia*), E. S. Sedov (*Vladimir, Russia*), I. V. Iorsh (*St. Petersburg, Russia*), M. Charukhchyan (*Vladimir, Russia*), A. V. Kavokin (*Southampton, United Kingdom*)

Cosmological Models with Nonlinear Metamaterials

16⁰⁰ – 16³⁰ **Alexander Plachenov** (*Moscow and St. Petersburg, Russia*), Alexei P.

Kiselev (*St. Petersburg, Russia*), Galina Dyakova (*St. Petersburg, Russia*)

Higher modes of the axisymmetrical waveguide with quadratic refraction index

16³⁰ – 17⁰⁰ *Coffee*

Young Researchers Symposium-2 (YRS-2)

Chairman: P. Stovicek

17⁰⁰ – 17¹⁰ **Dmitry Vavulin** (*St. Petersburg, Russia*)

Numerical solution of Shrodinger equation for biphoton wave function in twisted waveguide arrays

17¹⁰ – 17²⁰ **Alena Ivanova** (*St. Petersburg, Russia*)

Using of beam splitter and fiber splitter in quantum random number generators, based on fluctuations of vacuum

17²⁰ – 17³⁰ **Varvara Dubrovskaja** (*St. Petersburg, Russia*)

Temperature dependence of the optical fiber cable parameters in quantum cryptography systems

17³⁰ – 17⁴⁰ **Anton Kozubov** (*St. Petersburg, Russia*)

Theoretical investigation of the correlation between perturbations of linear optical CNOT gate parameters and its performance.

September 11, 2015

Chairman: A.Mantile

10⁰⁰ – 11⁰⁰ **Silvestro Fassari** (*Locarno, Switzerland*), Sergio Albeverio (*Bonn, Germany*),
Fabio Rinaldi (*Rome, Italy*)

Spectral properties of an isotropic three-dimensional quantum dot with a symmetric configuration of two identical attractive δ -impurities

11⁰⁰ – 11³⁰ *Coffee*

Chairman: S.Fassari

11³⁰ – 12⁰⁰ **Alexei D. Kiselev** (*St. Petersburg, Russia*)

Light scattering of Laguerre-Gaussian beams: Near-field structures and symmetries

12⁰⁰ – 12³⁰ **Muhammad Junaid Iqbal Khan** (*Multan, Pakistan*)

Topological effects in Graphene-like Nanosystems

12³⁰ – 13⁰⁰ **Igor Lobanov** (*St. Petersburg, Russia*)

Optical network simulation of time travel

13⁰⁰ – 13³⁰ **George Miroshnichenko** (*St. Petersburg, Russia*)

Quantum tomography of the quadratures of the microwave mode

13³⁰ – 13⁵⁰ **Irina Blinova** (*St. Petersburg, Russia*), Vadim Adamyan (*Odessa, Ukraine*),
Igor Popov (*St. Petersburg, Russia*)

Modeling of chain impurity on nanoparticle surface and catalytic activity.

13⁵⁰ – 14⁰⁰ *Closing*

Abstracts

Nudged Elastic Band method for systems with constraints

Evgeniy Alexandrov

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Nudged Elastic Band (NEB) is a well-developed method to locate the minimum energy path (MEP) between stable states on energy surface of a system. In some cases, however, there are constraints on values of parameters, which determine the energy surface. For example in magnetic systems, the energy surface is a function of magnetic moments. However, the magnitudes of moments often supposed to be constant. It needs modification of NEB for building of MEP. Variant of NEB for systems with constraints is suggested: each component in a compound system is processed with accordance to the corresponding manifold in configuration space. NEB procedure is implemented using translations for R-constrained components (e.g. particle coordinates), and rotations for S-constrained components (e.g. magnetic momentum vectors). As an example, the procedure is applied to magnetic spin systems and numerically approximated Alanine Dipeptide system.

Cosmological Models with Nonlinear Metamaterials

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We propose a novel mechanism for designing quantum hyperbolic metamaterials with use of semiconductor Bragg mirrors containing periodically arranged quantum wells. The hyperbolic dispersion of exciton-polariton modes is realized near the top of the first allowed photonic miniband in such structure. Exciton-light coupling provides a resonant non-linearity which leads to non-trivial topologic solutions. We predict formation of low amplitude spatially localized oscillatory structures: oscillons described by kink shaped solutions of the effective Ginzburg-Landau-Higgs equation. The oscillons have direct analogies in the gravitational theory. We discuss implementation of exciton-polariton Higgs fields for the Schrodinger cat state generation.

Selfadjoint realizations of the Laplacian on bounded Lipschitz domains

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In this talk we give a complete description of the selfadjoint realizations of the Laplacian on bounded Lipschitz domains with the help of Dirichlet and Neumann boundary conditions. One of the key difficulties is to establish the existence and the mapping properties of the Dirichlet and Neumann trace map on the domain of the maximal operator. We pay special attention to Robin type boundary conditions and we also discuss less standard realizations of the Laplacian, as e.g. the Krein-von Neumann extension and its spectral asymptotics.

This talk is based on joint work with Fritz Gesztesy and Marius Mitrea.

Modeling of chain impurity on nanoparticle surface and catalytic activity

Irina Blinova¹, Vadim Adamyan², Igor Popov¹

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²Odessa I.I.Mechnikov National University, Odessa, Ukraine

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Spectral problem for Schrodinger operator of half-crystal with surface impurities is considered. We use zero-range potentials model based on the theory of self-adjoint extensions of symmetric operators. The impurities are one-periodic chains of point-like potentials. The impurity leads to appearance of additional bands. The corresponding states are concentrated near the chain, i.e. it looks like a waveguide state. Hence, the electron density near the nanoparticle surface increases. This effect results in increasing of the catalytic activity of the nanoparticle.

Boundary triples approach to extensions of operator tensor products

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A model of two particles quantum system with the Hamiltonian having a form of a sum of tensor products is considered. Boundary triplet is constructed for the case when one operator assumed to be self-adjoint. The application of the suggested technique to a system of 1D particles is described.

Singular limit of a Schrödinger equation with nonlocal scaled short-range nonlinearity

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I will consider a model Schrödinger equation in dimension three with a singular nonlinear term concentrated in one point. This is a nonlinear version of the Schrödinger equation with a point interaction (or point-potential). It was studied in a series of papers by R. Adami, G. Dell'Antonio, R. Figari, and A. Teta, and it is known how to describe a unitary flow on constant energy surfaces. I will show that such model can be understood as an effective one for a regular Schrödinger equation with a nonlocal scaled short-range nonlinearity as the scaling parameter goes to zero, and discuss the comparison with the one dimensional case. Models with nonlinearities concentrated in one point (in one and three dimensions) are well known and extensively studied in mathematical physics. Our result represents a first attempt to justify such models in dimension three, in terms of the approximation through a regularized dynamics. This is a joint work in collaboration with D. Finco, D. Noja, and A. Teta

Strong delta-interaction supported by a surface

Jaroslav Dittrich

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Delta interaction supported by a 2-D compact surface with boundary in a 3-D space is considered. The asymptotics of the low eigenvalues at the strong coupling is given. With P. Exner, K. Pankrashkin, Ch. Kuhn.

Temperature dependence of the optical fiber cable parameters in quantum cryptography systems

Varvara Dubrovskaja

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The problem of protecting confidential information transferred by public telecommunication channels nowadays is of great interest, due to new possibilities for decrypting information. One way to solve this problem is to use quantum cryptography (QKD) systems which exploit the unique properties of single photons. Several practical QKD systems have been experimentally demonstrated during the last two decades, among them the subcarrier wave quantum cryptography system, which remains promising for integration into fiber optic communication lines. In practical QKD several technological problems still remain, including synchronization of the receiver and transmitter devices. The problem arises from necessity of precisely controlling the phase of high-frequency electrical modulating signals using optical signals which decay and distort during transmission through the fibers. Current research is focused on temperature dependence of signals in an optical fiber cable. The temperature model was created in order to determinate signal delay in the cable, and optimizes parameters of the calibration procedure of the system. It has been shown that system adjustment every 158 ms is required for stable operation of the subcarrier wave quantum cryptography system, which is three times more than previously used. Therefore, it allows increasing QKD key generation rate.

Spectral properties of an isotropic three-dimensional quantum dot with a symmetric configuration of two identical attractive δ -impurities

Sergio Albeverio^{1,2,3,4}, Silvestro Fassari^{2,5,6} and Fabio Rinaldi^{2,6,7}

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In this presentation we wish to provide an overview of the spectral features of the self-adjoint Hamiltonian of the three-dimensional isotropic harmonic oscillator perturbed by either a single attractive δ -interaction centred at the origin or by a pair of identical attractive δ -interactions symmetrically situated with respect to the origin.

Given that such Hamiltonians represent the mathematical model for quantum dots with sharply localised impurities, we cannot help having the renowned article by Brüning, Geyler and Lobanov [1] as our key reference. We shall also compare the spectral features of the aforementioned three-dimensional models with those of the self-adjoint Hamiltonian of the harmonic oscillator perturbed by an attractive δ' -interaction in one dimension fully investigated in [2,3] given the existence in both models of the remarkable spectral phenomenon called "level crossing". The rigorous definition of the self-adjoint Hamiltonian for the singular double well model will be provided through the explicit formula for its resolvent (Green's function). Furthermore, by studying in detail the equation determining the ground state energy for the double well model, it will be shown that the concept of "positional disorder" introduced in [1] in the case of a quantum dot with a single δ -impurity can be extended also to the model with the twin impurities in the sense that the greater the distance between the two impurities is, the less localised the ground state will be. Another noteworthy spectral phenomenon will also be observed: if the distance between the two centres falls below a certain threshold value, it will be possible to have a range of values of the strength of the twin point interactions for which the first excited symmetric bound state is more tightly bound than the lowest antisymmetric bound state. Furthermore, it will be pointed out that the Hamiltonian with the singular double well, which requires the renormalisation technique to be defined, does not converge, as the distance between the two impurities shrinks to zero, to the one with a single δ -interaction centred at the origin having twice the strength in contrast with what happens to its one-dimensional analogue for which no renormalisation is required. It is worth stressing that this phenomenon has been recently observed also in the case of another model requiring the renormalisation of the coupling constant, namely the one-dimensional Salpeter Hamiltonian perturbed by two twin attractive δ -interactions symmetrically situated at the same distance from the origin.

Topological effects in Graphene like Nanosystems

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The interplay of topology and geometry plays a vital role in studying properties of various materials. Physically it is very interesting in the sense that involves magnetic field that could count the quantum Hall effect. Under the action of perpendicular magnetic field, the surface of materials in various dimensions can transform to surface in one reduce dimenson. The resulting geometry could be a torus, sphere, circle or any irregular or regular shaped object. The shape of the object define the physical properties of the materials in material sciences. It involves spin-orbit interaction and time-reversal symmetry.

Using of beam splitter and fiber splitter in quantum random number generators, based on fluctuations of vacuum

A. Ivanova

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Quantum random number generators are based on non-deterministic physical processes. They allow one to obtain sequences of random numbers, that can be used in applications requiring high degree of randomness. In quantum random number generation schemes, based on quantum fluctuations of the vacuum, beam splitters with two inputs and two outputs are normally used. The purpose of this research was a comparison of quantum descriptions of such beam splitter and a fiber splitter with one input and two outputs. In this research quantum mathematical descriptions of using Y-splitter and beam splitter in quantum random generation schemes based on fluctuation of vacuum were obtained. For beam splitter we described relationship between input radiation and differential current for quantum random number generation scheme using homodyne detection. We also derived expressions allowing estimation of scheme parameters imperfection impact on measurement results. A comparison of results showed that for two types of optical splitters mathematical quantum description of resulting output signals is identical. That allows using fiber splitter with one input and two output ports at quantum random number generator based on fluctuations of the vacuum, instead of optical beam splitter.

Light scattering of Laguerre-Gaussian beams: Near-field structures and symmetries

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This study deals with the characteristics of light scattering from spherical dielectric particles illuminated by a Laguerre-Gaussian (LG) light beam. We apply the method of far-field matching to remodel the beam and use the theoretical results to analyze the optical field in the near-field region for purely azimuthal LG beams characterized by a nonzero azimuthal mode number. The morphology of photonic nanojets and the near-field structure of optical vortices associated with the components of the electric field are both found to be highly sensitive to the mode number. We also discuss the symmetry properties of laser beams and related results for the optical forces.

Theoretical investigation of the correlation between perturbations of linear optical CNOT gate parameters and its performance

Anton Kozubov

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This paper is devoted to a new method for finding the operability of some quantum optical circuits which depends on their constructions, such as reflectivity of beam splitters in the linear optical controlled-NOT gate in the coincidence basis.

Effect of Stone-Wales defects on the thermal conductivity of graphene

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The problem of phonon scattering by strain fields caused by Stone-Wales (SW) defects in graphene is studied in the framework of the deformation potential approach. An explicit form of the phonon mean free path due to phonon-SW scattering is obtained within the Born approximation. The mean free path demonstrates a specific q -dependence varying as q^{-3} at low wavevectors and taking a constant value at large q . The thermal conductivity of graphene nanoribbons (GNRs) is calculated with the three-phonon umklapp, SW and rough edge scatterings taken into account. A pronounced decrease of the thermal conductivity due to SW defects is found at low temperatures whereas at room temperatures and above the phonon-phonon umklapp scattering becomes dominant. A comparison with the case of vacancy defects shows that they play more important role in the reduction of the thermal conductivity in GNRs over a wide temperature range.

Optical network simulation of time travel

Igor Lobanov

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Quantum optical networks in four dimensions are considered to roughly simulate Feynman diagrams. Such a network containing a closed loop in space-time is studied using Deutsch's approach. The obtained model of time travel is simple enough to be explicitly solvable. Introducing dependence of network elements on a controllable parameter we discuss opportunities to convey a message back in time.

Benchmark solutions for Stokes flow through nanostructures

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Numerical algorithms are widely used for modeling flows through nanostructures. One of the most effective method to verify a numerical algorithms is a comparison with a benchmark solution. We obtain exact particular solutions of Stokes equations with variable viscosity in spherical coordinates for a case of spherically symmetric viscosity. This solutions can be used as a benchmark for algorithm testing. We have developed implementation of multigrid method for solving Stokes equations with variable viscosity in spherical coordinates. The results of flow computation by this method were compared with the benchmark solutions.

Selfadjoint elliptic operators with boundary conditions on not closed hypersurfaces

Andrea Mantile

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The abstract theory of self-adjoint extensions of symmetric operators is used to construct self-adjoint realizations of a second-order elliptic differential operator on R^n with linear boundary conditions on (a relatively open part of) a compact hypersurface. Our approach allows to obtain Krein-like resolvent formulae where the reference operator coincides with the "free" operator with domain $H^2(R^n)$; this provides an useful tool for the scattering problem from a hypersurface.

Moreover, Schatten-von Neumann estimates, for the difference of the powers of resolvents of the free and the perturbed operators, yield the existence and completeness of the wave operators of the associated scattering systems. As a concrete example, we focus on Robin-type boundary conditions on a $(n-1)$ -dimensional sheet with regular boundary.

This is a joint work with: A. Posilicano and M. Sini

Quantum tomography of the quadratures of the microwave mode

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Cavity CED experiments in which as an isolated quantum system are the photons of microwave mode of high-Q resonator with superconducting walls, are of particular interest for quantum information science. Methods of preparation of various quantum states of photons, such as Fock states, squeezed states, Schrödinger cat - states proposed and implemented in the works [1 - 7]. Quantum states of the photon mode are described in terms of quasi-distributions in the phase space. The task of reconstruction quasi-distributions in quantum measurement is of particular importance. Alternative to [8], the method to reconstruct the density matrix of a mode created in the high-Q resonator, is proposed in this report. A method of measuring the probability density $P(x(\varphi)) = \langle x(\varphi) | \rho | x(\varphi) \rangle$ of a field quadrature for different values of phase are presented here. It is shown that such measurement is possible in a more simple installation that does not require fine tuned atomic interferometer and additional classic source of coherent radiation. The phase φ installation is performed using a rotation basis after the departure of atoms-probes from the resonator. The probability of the difference $\Delta = N_+ - N_-$ of numbers of atoms emitted in an excited and ground states in a sample of N flying through the cavity atoms - probes one behind the other is measured value. It is shown that if the interaction time τ of the probe and a cavity is sufficiently small, as measured the probability equal the probability of $P(x(\varphi))$, where the desired value of $x(\varphi)$ is defined as

$x(\varphi) = \frac{N_+ - N_-}{2N\tau}$. Numerical simulations performed for the numerical values of the

parameters corresponding to the [9 - 11] shows the effectiveness of the proposed method.

This work was financially supported by the Government of Russian Federation (grant 074-U01) and by the Ministry of Science and Education of the Russian Federation (project 14.Z50.31.0031)

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Resonance mechanism of Earthquakes

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The "M8 Global test algorithm" for earthquake prediction (1984) is based on the observation that almost 80 % of actual events at the selected location arise due to the stress built up thanks to previous events at the corresponding Earthquake-prone (active) zone, stored in the form of static elastic deformation and displacement of tectonic plates in the gravitational field. Though M8 algorithm is extremely efficient providing higher than 99 % confidence level for prediction of the Time intervals of Increased Probability (TIP) of the Earthquakes, yet some of highly dangerous events, like recent Tohoku earthquake (Japan, March 11, 2011) were not predicted. In Tohoku the "black box" constructed based on the above algorithm, removed the TIP warning from the list of expected earthquakes at the Tohoku location 70 days before the earthquake. In our talk we suggest a model taking into account also the dynamical component of the elastic energy stored in the form of seismo-gravitational oscillations (SGO) on the tectonic plates and propose the hypothesis on resonance triggering of the earthquakes due to beats of the SGO modes with close frequencies, localized on the active zone of the expected earthquake and (delocalized) on the complement. Using a solvable model of the tectonic plate with relatively small active zone, we estimate the energy transfer between these modes, defined by the beats of the modes.

Higher modes of the axisymmetrical waveguide with quadratic refraction index

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We consider Gaussian-beam-type solutions of the parabolic equation, which describe paraxial propagation of localized waves in the axisymmetrical waveguide. We seek solutions in the form $U=Hg$, where g is the fundamental Gaussian mode, and H is a non-constant function, which we call amplitude. We show that in specially chosen coordinates the amplitude satisfy secondary parabolic equation for homogeneous medium. In particular, we consider Laplace-Gaussian and Helmholtz-Gaussian modes which generalize analogous solutions found earlier by A.P.Kiselev for the case of a homogeneous medium.

Limiting absorption principle and factorized formulae for self-adjoint Laplace operators with boundary conditions on hypersurfaces

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We provide a Limiting Absorption Principle for the self-adjoint realizations of the Laplacian corresponding to boundary conditions on (relatively open parts of) compact hypersurfaces. For any of such self-adjoint operators we also provide an explicit characterization of the scattered field by a factorized formula depending on the incident wave. Both such results are obtained by a Krein-type formula which gives the resolvent difference between the free Laplacian and the ones corresponding to boundary conditions on the hypersurface.

Joint work with Andrea Mantile and Mourad Sini.

On the discrete spectrum of the Dirac operator on the branching chain quantum graph

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In this work we considered the Dirac operator acting on the quantum graph presented by the chain of rings with Y-branching. The dispersion equations are obtained in an explicit form. Also, we have a conditions whether the roots of the dispersion equations belong to the discrete spectrum. These equations and conditions were analyzed numerically.

A construction of the propagator for two Aharonov-Bohm solenoids in a uniform magnetic field

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An invariant quantum Hamiltonian $H = -\Delta_{LB} + V$ is considered in the L_2 space based on a Riemannian manifold \tilde{M} with a discrete symmetry group Γ . To any unitary representation Λ of Γ one can relate another operator on $M = \tilde{M} \setminus \Gamma$, called H_Λ , which formally corresponds to the same differential operator as H but which is determined by quasi-periodic boundary conditions. As originally observed by Sunada in mathematics and Schulman in theoretical physics, one can construct the propagator associated with H_Λ , provided one knows the propagator associated with H . This approach has already been applied earlier to a quantum model describing a charged particle on the plane with two Aharonov-Bohm vortices. This contribution aims to show that this result can be extended to the case when the plane is subject to a uniform magnetic field.

Numerical solution of Schrodinger equation for biphoton wave function in twisted waveguide arrays

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We consider generation of entangled biphoton states with orbital-angular-momentum in triangular quadratic waveguide arrays with twisted geometry. For this purposes we derive Shrodinger equation for biphoton wave function and equation for pump field profile. We describe numerically the process of biphoton generation through spontaneous four-wave mixing. We suggest that biphoton correlations can be controlled by the amount of twist and pump field profile.

Size quantization and the resonance concept of the low-threshold emission

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The 1D Fowler - Nordheim model of resonance transmission of electron from the carbon-covered cathode to vacuum provides a realistic estimation for the corresponding transmission coefficient, but fails to estimate the emission current, which requires the preliminary calculation of the corresponding dispersion on the corresponding spectral band. We calculated the dispersion based on Dirichlet - to-Neumann map of the period of the periodic lattice formed on the interface of the metallic cathode and the carbon cover. Our approach developed for the 2D periodic lattice, is general and may be used for calculation of the dispersion for multidimensional periodic lattices.