Problem 1. Are there continuously differentiable functions f, g such that they are not constant on some interval and the following relation takes place at each point of this interval:

$$(f(x)g(x))' = f'(x)g'(x)?$$

Problem 2. Let $\omega_1, \ldots, \omega_{2016}$ be different roots of degree 2016 of 1. Calculate $\prod_{k\neq l}(\omega_k - \omega_l)$.

Problem 3. Prove that

$$I = \int_0^\infty ... \int_0^\infty \frac{dx_1...dx_n}{1 + x_1^{p_1} + ... + x_n^{p_n}} > 1$$

for any positive numbers $p_1...p_n$ such that the integral converges.

Problem 4. Calculate the exact value of the integral

$$\int_0^{+\infty} 2^{-x} \frac{2^{x-1} - 1 + 2^{-x-1}}{x^2} dx.$$

Problem 5. Let f be smooth $2\pi-$periodic function. For any segment I of length $|I|$, we determine $f_I = \frac{1}{|I|}\int_I f(t)dt$. For any f we introduce $\|f\| = \sup_I \frac{1}{|I|}\int_I |f(t) - f_I|dt$. Let I and J, $I \subset J$, be segments with the common middle point. Prove that

$$|f_I - f_J| \leq 2(\log_2 \frac{|J|}{|I|} + 1)\|f\|.$$

Problem 6. Let A be a positive definite symmetric real $n \times n$ matrix. Assume that all entries of A are non-negative. Let c_i denote the sum of entries in i-th row of A, $i = 1, \ldots, n$. Let k be the sum of all n^2 entries of the matrix A^{-1}. Prove that $k \geqslant 1/c_1 + 1/c_2 + \cdots + 1/c_n$.

Problem 7. For vectors $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n)$ in $\mathbb{C}^n$ we denote $\langle x, y\rangle = \sum_{i=1}^n x_i\bar{y}_i$. Prove that if $\langle x, x\rangle = \langle y, y\rangle = \langle z, z\rangle = 1$ then

$$\Re\left((1 - \langle x, y\rangle)(1 - \langle y, z\rangle)(1 - \langle z, x\rangle)\right) \geq 0.$$

(here $\Re a$ denotes the real part of a complex number a.)