PLANNING UNDER DIFFERENTIAL CONSTRAINTS

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GENERAL FRAMEWORK UNDER DIFFERENTIAL CONSTRAINTS

1. **Initialization:** Let G(V, E) represent an undirected search graph, for which the vertex set V contains a vertex x_I , and the edge set E is empty.
2. **Swath-point Selection Method (SSM):** Choose a vertex x_{cur} for expansion.
3. **Local Planning Method :** Generate a motion primitive such that $u(0)= x_{cur}$ and $u(t_F) = x_r$ for some $x_r \in X_{free}$, which may or may not be a vertex in G. Using the system simulator, a collision detection algorithm, and by testing the phase constraints, it must be verified to be violation-free. If this step fails, then go to Step 2.
4. **Insert an Edge in the Graph**
5. **Check for a Solution**
6. **Return to Step2:** Iterate unless a solution has been found or some termination condition is satisfied

SEARCHING ON A LATTICE

Grid search techniques to motion planning.

The difficulty – to choose a discretization that leads to a lattice that can be searched using any of the search techniques.

A DOUBLE-INTEGRATOR LATTICE

Let $C = C_{free} = R$ and $\ddot{q} = u$.

The phase space is $X = R^2$, and $x = (q, \dot{q})$.

Let $U = [-1, 1]$.

$\dot{q}(t) = \dot{q}(0) + ut,$

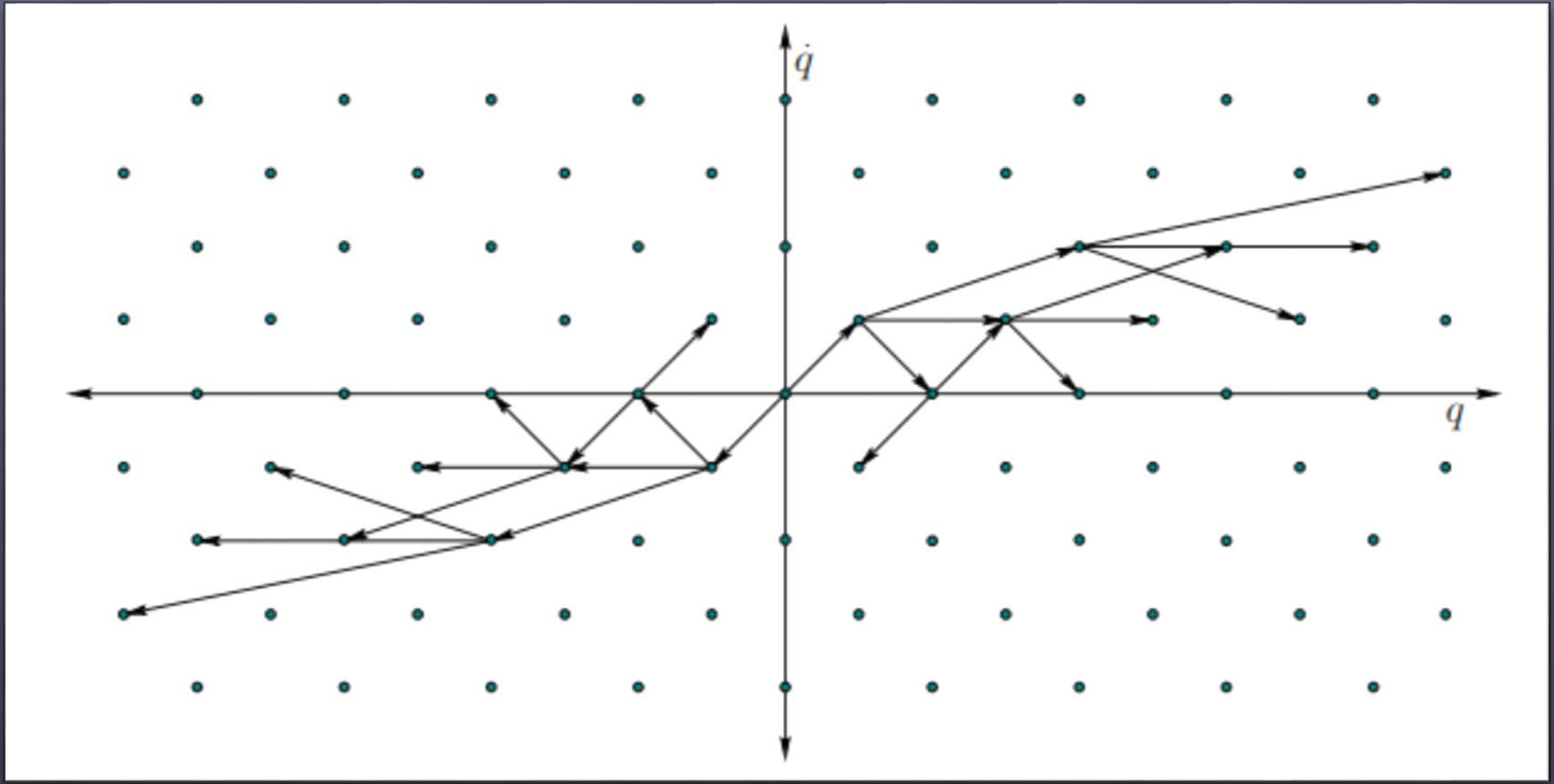
$q(t) = q(0) + \dot{q}(0)t + \frac{1}{2} ut^2$

$U_d = \{-1, 0, 1\}$

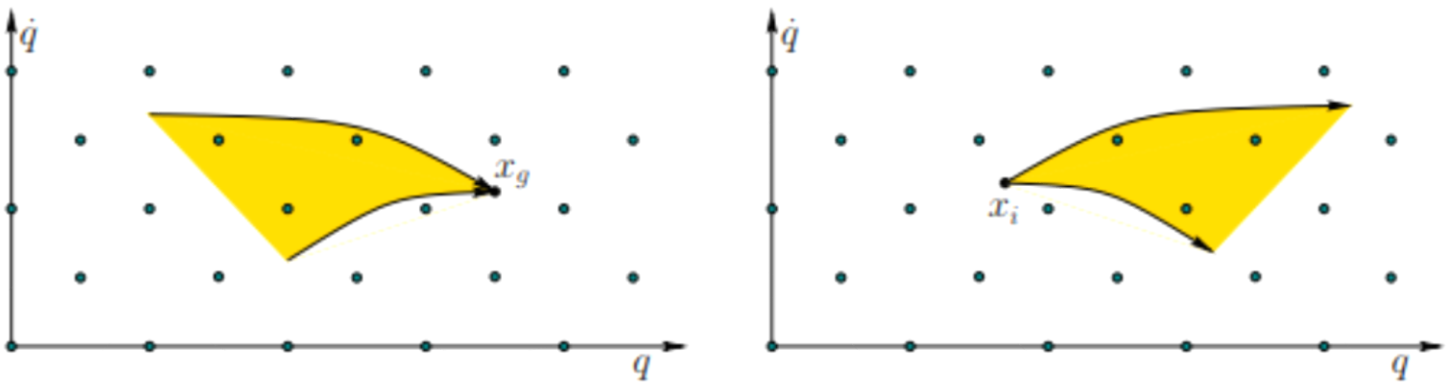
$x_{k+1} = f_d(x_k, u_k),$

$q_k = q_1 + \frac{i}{2} (\Delta t)^2$

$\dot{q}_k = \dot{q}_1 + j\Delta t,$



$\dot{q}$
q



(a) Backward reachable set from x_G (b) Forward reachable set from x_I

Let $v_{max} > 0$ be a positive constant and assume that $|\dot{q}| \leq v_{max}$

EXTENSION

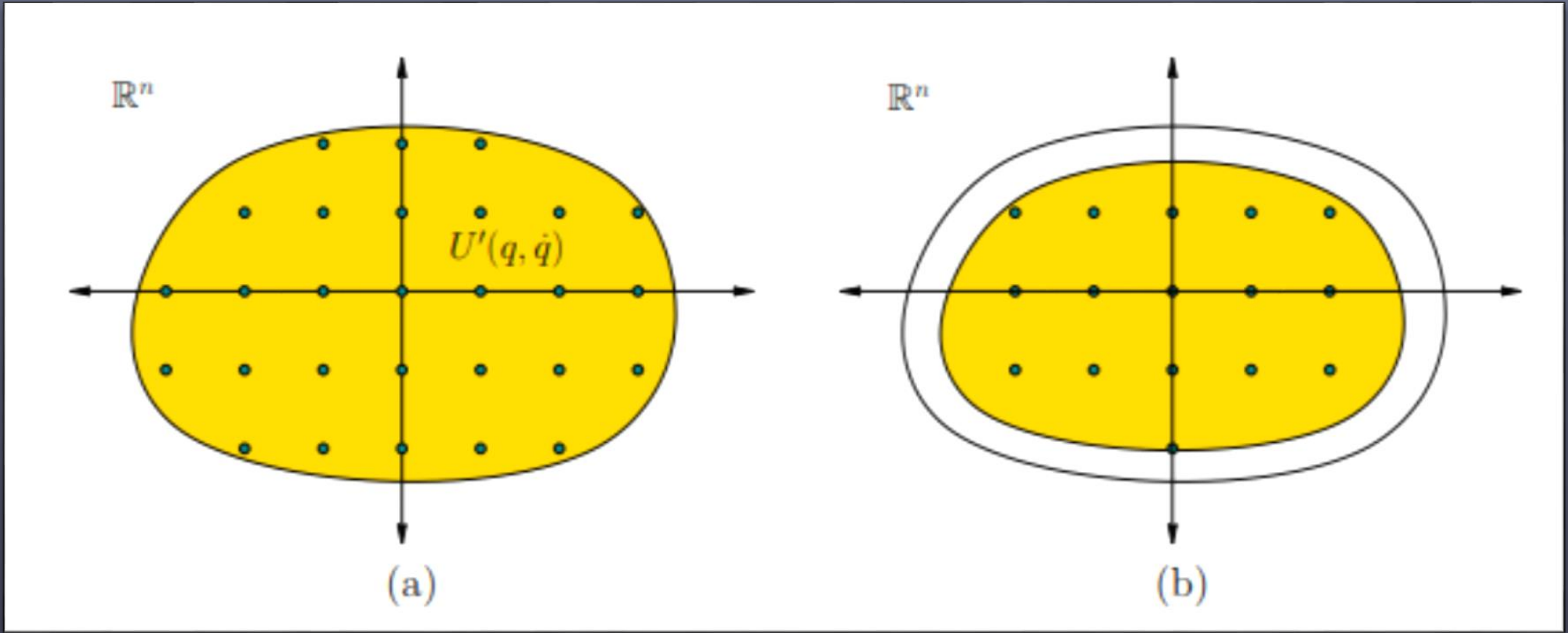
- $C = R^n$ and each $q \in C$ is an n-dimensional vector
- There are n action variables and n double integrators of the form $\ddot{q}_i = u_i$, $U_i = [-1, 1]$
- The phase space X is R^{2n} , and each point is $x = (q_1, . . . , q_n, \dot{q}_1, . . . , \dot{q}_n)$
- $\dot{x}_i = x_{n+i}$ and $\dot{x}_{n+i} = u_i$

UNCONSTRAINED MECHANICAL SYSTEMS

- $U'(q, \dot{q}) = \{\ddot{q} \in R^n \mid \exists u \in U$ such that $\ddot{q} = h(q, \dot{q}, u)\}$
- The main differences are

1. The set $U'(q, \dot{q})$ may describe a complicated region in R^n, whereas U in the case of the true double integrators was a cube centered at the origin.

2. The set $U'(q, \dot{q})$ varies with respect to q and $\dot{q}$. Special concern must be given for this variation over the time sampling interval Δt. In the case of the true double integrators, U was fixed



$\mathbb{R}^n$
$U'(q, \dot{q})$
(a)
$\mathbb{R}^n$
(b)

INCORPORATING STATE SPACE DISCRETIZATION

- For underactuated and nonholonomic systems
- X can be partitioned into small cells, within which no more than one vertex is allowed in the search graph

ALGORITHM

CELL-BASED SEARCH(x_I , x_G)

Q.insert(x_I);

G.init(x_I);

while Q ≠ ∅ and x_G is unvisited

    $x_{cur} \rightarrow$ Q.pop();

    for each (u_t , x) ∈ reached(x_{cur})

        if x is unvisited

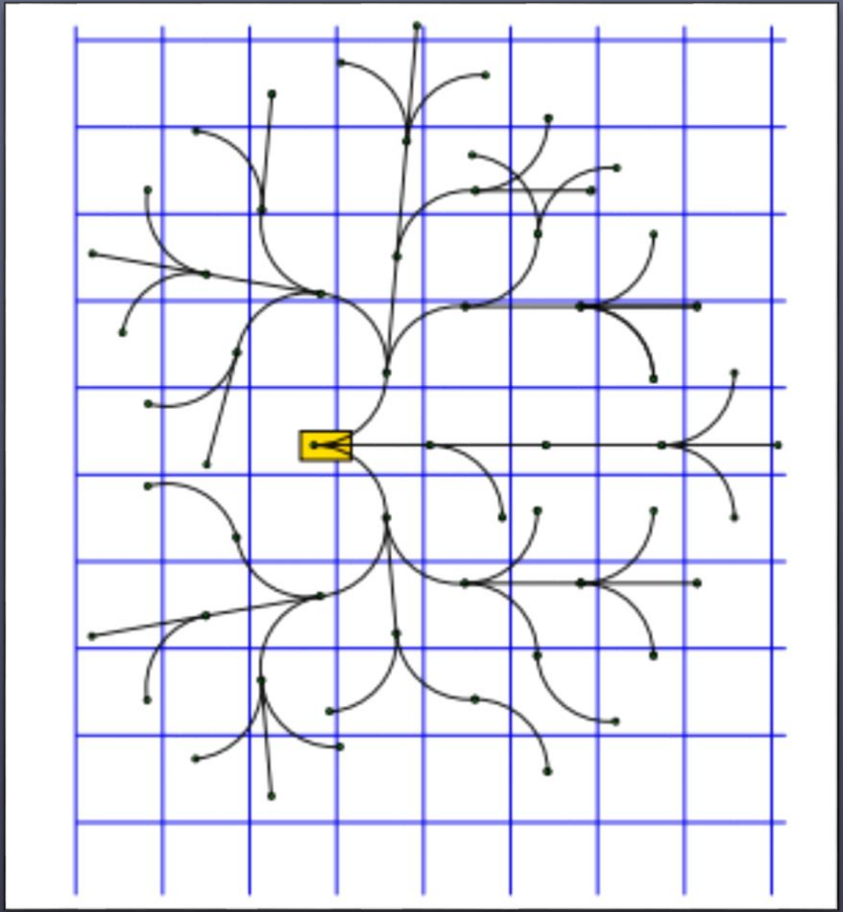
            Q.insert(x);

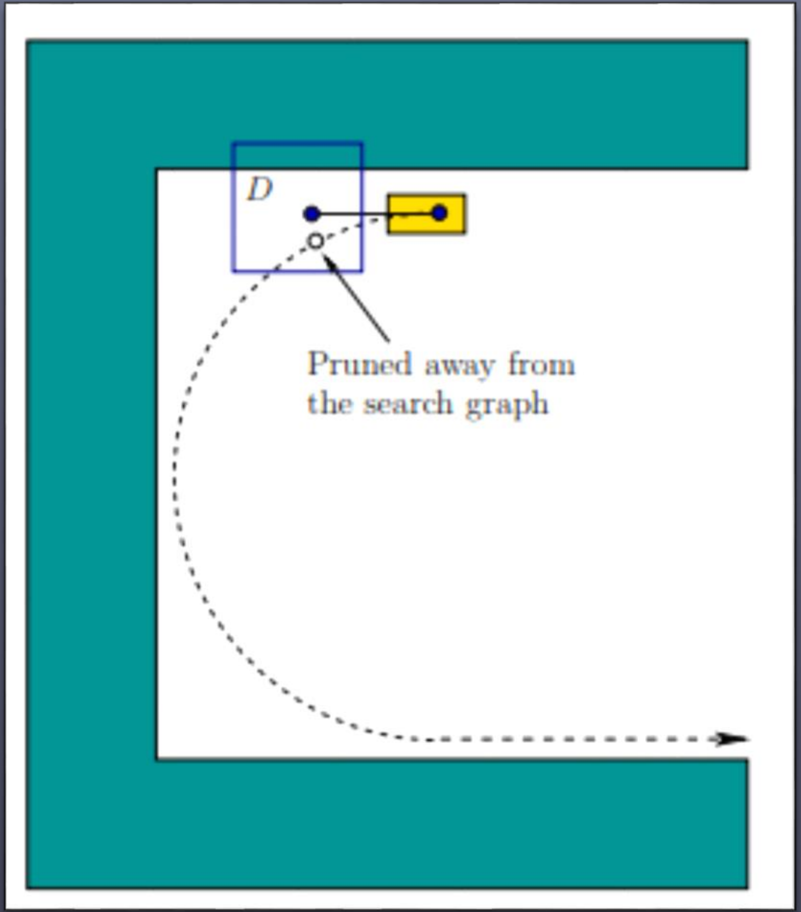
            G.add vertex(x);

            G.add edge(u_t);

            Mark cell that contains x as visited;

Return G;




D
Pruned away from
the search graph

СПАСИБО ЗА ВНИМАНИЕ!